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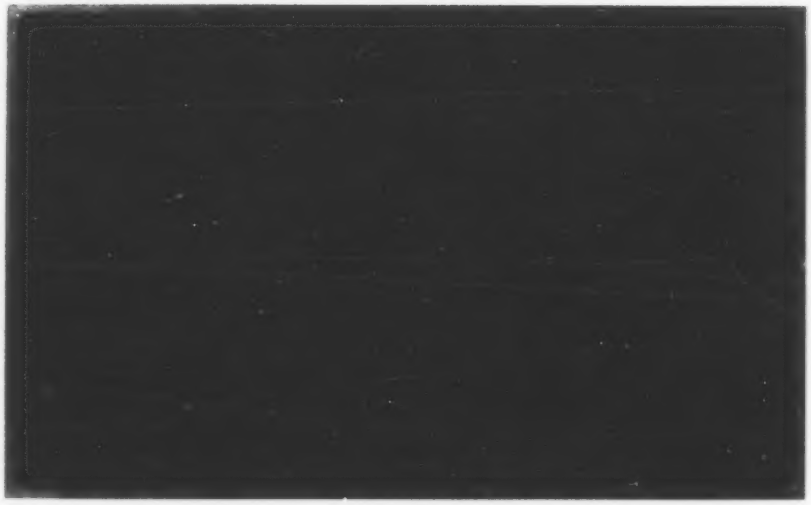

CENTRE DE RECHERCHE SUR LES TRANSPORTS
UNIVERSITÉ DE MONTRÉAL
C.P. 6128, SUCCURSALE A
MONTRÉAL
CANADA H3C 3J7

TÉLÉPHONE : (514) 343-7575
EMAIL : crt@crt.umontreal.ca
TÉLÉCOPIEUR (FAX) : (514) 343-7121

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A Tabu Search Heuristic for the Vehicle Routing Problem with Time Windows

par

Jean-Yves Potvin
Tanguy Kervahut
Jean-Marc Rousseau

Centre de recherche sur les transports, Université de Montréal, C.P. 6128, succursale A, Montréal, Canada H3C 3J7.

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Section 1. Introduction

A tabu search heuristic is proposed in this paper for solving the Vehicle Routing Problem with Time Windows (VRPTW). The VRPTW has been intensively studied in the literature and the reader can find reviews of the main problem-solving approaches in this domain in [Bodin et al. 83, Desrochers et al. 88, Solomon and Desrosiers 88].

The objective of the VRPTW is to service a set of customers at minimum cost with a fleet of vehicles of finite capacity housed at a central depot. Each customer has a known demand to be serviced (either for pick-up or delivery but not both) and a time window for that service. A time window is also associated to the depot in order to define a "scheduling horizon". Each route must then start and end within the bounds of that horizon.

Since we consider the hard time window case, a vehicle cannot service a customer if it arrives too late at the customer location.

Résumé. Cet article décrit une recherche taboue pour le problème de tournées de véhicules avec fenêtres de temps. La recherche taboue fait appel à une heuristique d'échange conçue spécifiquement en fonction des fenêtres de temps. Des résultats numériques sur l'ensemble des problèmes tests de Solomon sont présentés à la fin de l'article.

Mots Clés. Recherche Taboue, Tournées de Véhicules, Fenêtres de Temps.

Abstract. This paper describes a tabu search heuristic for the Vehicle Routing Problem with Time Windows (VRPTW). The tabu search incorporates an exchange heuristic which is specifically designed for problems with time windows. Computational results on the standard set of problems of Solomon are included at the end of the paper.

Key Words. Tabu Search, Vehicle Routing, Time Windows

Moreover, the vehicle must wait if it arrives too early. Hence, the total routing and scheduling costs include not only the total travel distance (or total travel time), but also the waiting time.

Both exact and heuristic algorithms have been designed for solving that problem. An important development relating to exact algorithms for the VRPTW is described in [Desrochers et al. 92]. Optimum solutions for some 100-customer problems have been obtained with a column generation algorithm. However, in order to achieve such results, the branching and bounding procedure within the algorithm must be tuned for the problem (or class of problems) under consideration.

On the other hand, heuristic problem-solving approaches for the VRPTW are still very useful because they can solve much larger problems in much less computation time than exact algorithms. Route building as well as route improvement heuristics have been described in the literature for the VRPTW. For route building procedures, we refer in particular to [Solomon 87, Potvin and Rousseau 90a]. Description of route improvement heuristics for the VRPTW may be found in [Or 76, Solomon et al. 88, Potvin and Rousseau 90b, Savelsbergh 90].

In the following, we describe a tabu search heuristic based on the exchange heuristic of [Potvin and Rousseau 90b]. This exchange heuristic is first introduced in Section 2. Then, section 3 presents the tabu search, which is inspired from the work of [Gendreau et al. 91]. Finally, section 4 presents computational results as obtained on the set of problems of Solomon [Solomon 1987].

Section 2. An Exchange Heuristic for Problems with Time Windows

Starting from some initial solution, improvement heuristics consider various modifications to this solution in order to find a new improved solution. The procedure is then repeated until no more improvement is possible.

The best known heuristics in this class are the k-opt exchanges of [Lin 65]. Here, k links are removed from the current tour and replaced by k new links in order to generate a new tour. The 2-opt exchanges, in particular, are often used for improving routes. However, they are not well adapted to problems with time windows. This is related to the fact that they do not preserve route orientation. Figure 1 shows a 2-opt on a single route, where the links (1,2) and

(3,4) are replaced by the links (1,3) and (2,4). Since customers are typically sequenced according to their time window's upper bound (those with the lowest upper bounds being the first to be serviced), reversing some portion of a route is likely to generate a non feasible solution.

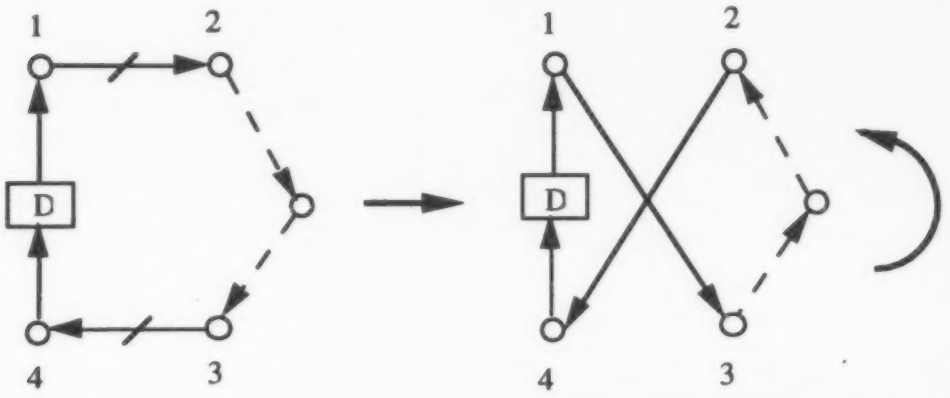

Fig. 1 Exchange of links (1,2),(3,4) for links (1,3),(2,4)

For problems with multiple routes, it is possible to look at another way of exchanging two links. First, the multiple routes are merged into a single route by creating one copy of the depot for each route and by connecting each copy to the other nodes in the same way as the original depot (figure 2a) [Svestka and Huckfeld 73, Bellmore and Hong 74]. We then define a special 2-opt* exchange, where two links are replaced by two new links so as to split the tour into two distinct subtours (figure 2b). The subtours are finally connected together to form an equivalent tour by exchanging the links connecting two customers to distinct copies of the depot (figure 2c).

The 2-opt* exchange is particularly efficient for problems with time windows, because it preserves route orientation by introducing the last customers of a given route (i.e. those with high upper bounds) at the end of the first customers of another route (i.e. those with low upper bounds). The new solution is thus likely to be feasible.

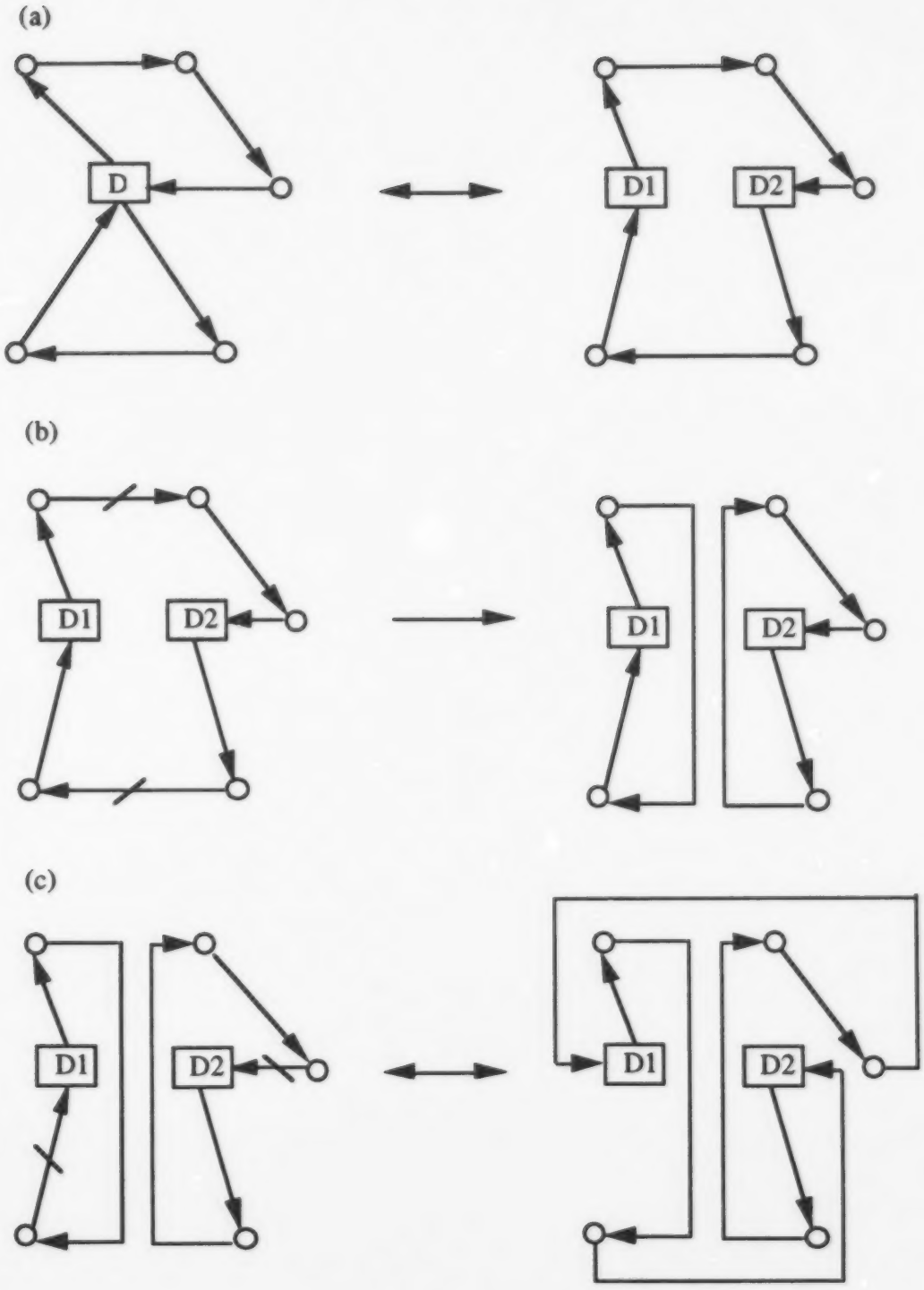

Fig. 2 A 2-opt* exchange

If we look at the tour before and after the 2-opt* exchange, four different links have been replaced, including the two links for connecting the two subtours together. The set of 2-opt* exchanges is thus a restricted subset of 4-opt exchanges that are particularly well suited to problems with time windows.

It should be emphasized here that the 2-opt* exchanges are appropriate for inter-route exchanges only. This strategy is not suitable when a single route is involved, because a subtour with no copy of the depot would then be generated. In order to get intra-route as well as inter-route improvements, we combine the above 2-opt* exchanges with the well-known Or-opt exchanges [Or 76]. The latter strategy considers in turn sequences of one, two and three adjacent customers in the current solution and try to place them at another location while preserving route orientation (figure 3). Those exchanges are a restricted subset of 3-opt exchanges that are particularly well suited to problems with time windows.

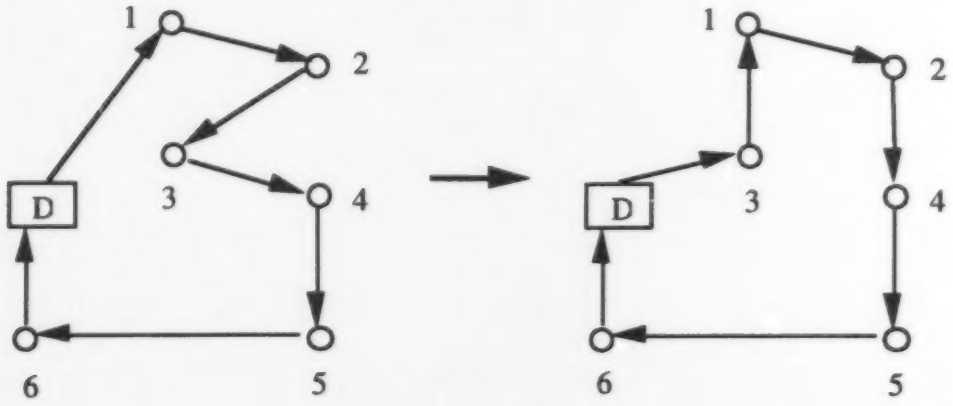

Fig. 3 Or-opt on customer 3 between depot D and customer 1

We have now described the two basic ingredients of our tabu search heuristic, namely the 2-opt* and Or-opt exchanges. The following section will now describe the tabu search as a whole.

Section 3. The Tabu Search Heuristic

The tabu search is an improvement heuristic introduced by Glover [Glover 89, Glover 90]. At each step, the neighborhood of the current solution is explored and the best solution in the neighborhood is selected as the new current solution. However, the procedure does not stop when no more improvement is possible. The best solution in the neighborhood is selected as the current solution even when it is worse than the current solution. Such a strategy allows the method to escape from local extrema and, as a consequence, to explore a larger part of the search space.

To prevent cycling, recently selected solutions are forbidden and inserted into a so-called "tabu list". Usually, the solutions are not stored as such in the tabu list. Rather, for each modification leading to a new solution, the inverse modification is labelled tabu and stored in the tabu list. For the 2-opt* and Or-opt exchanges, the modifications are defined in terms of the links that are removed from the current solution, and it is forbidden to insert recently removed links. Also, as in most tabu implementations, a tabu move can be overridden if an overall best solution is generated by doing so.

Our tabu search heuristic is largely inspired by the work of [Gendreau et al. 91] for the Vehicle Routing Problem. In particular, we associate a random tabu tag to each forbidden move. The move is then declared tabu for the number of iterations indicated by the tag. It is a way to implement a tabu list with variable length, so as to increase the probability of obtaining a global optimum [Taillard 90].

A major difference with the work of Gendreau et al. is that only modifications that maintain feasibility of the current solution are allowed. As a matter of fact, it seems to be quite difficult in a VRPTW context to get back to a feasible solution, once a solution with time window violations is accepted.

Our heuristic, in its current form, is stated below. It is important to note in that respect that the solutions are evaluated first with respect to the number of routes and, secondly, with respect to total route time. Hence, a solution with M routes is better than a solution with M+1 routes, independently of the total route time.

Tabu Search Heuristic

Step 1. Initialization step. For every customer, compute the h nearest neighbors in distance. Then, generate an initial feasible solution with Solomon's heuristic, and set it as the current solution. Evaluate the number of routes M and the total route time RT (travel time + waiting time + service time).

Repeat Step 2 until there is no improvement for I_{tot} consecutive iterations:

Step 2. Tabu search step.

Repeat Step 2' until there is no improvement for I_{2opt*} consecutive iterations

Step 2'. Define the neighborhood of the current solution by selecting q customers at random in the tour. For each selected customer i, consider all possible 2-opt* exchanges that maintain feasibility and satisfy the following conditions:

> if links (i, i+1) and (j,j+1) are to be replaced by links (i,j+1) and (j,i+1), then the first two links must be in different routes and customer j+1 must be one of the h nearest neighbors of customer i.

Then, the best solution in the neighborhood with respect first to the number of routes M and secondly, with respect to total route time RT, is selected as the current solution. The inverse move is declared tabu for I_{tabu} iterations, where I_{tabu} is a random number in the interval [5,10].

Repeat Step 2" until there is no improvement for I_{or} consecutive iterations.

Step 2". It is the same as Step 2', except that Or-opt exchanges are now applied. Here, for each selected customer i, we consider all possible Or-opt exchanges that maintain feasibility and satisfy the following condition:

if customer i (with possibly customer i+1 and i+2) is to be moved between customer j an j+1, then customer j must be one of the h nearest neighbors of customer i.

Step 3. The best feasible solution found during the search is the final result of the tabu heuristic.

In the next section, we present the computational results as obtained on Solomon's standard set of problems.

Section 4. Computational Results

We applied the above tabu search heuristic on the set of problems of Solomon, which are all 100-customer euclidean problems. The geographical data were either randomly generated using a uniform distribution (problem sets R1 and R2), clustered (problem sets C1 and C2) or semi-clustered with a mix of randomly distributed and clustered customers (problem sets RC1 and RC2). Problem sets R1, C1 and RC1 have a short scheduling horizon, and allow only a few customers per route. Conversely, problem sets R2, C2 and RC2 have a long scheduling horizon and allow a larger number of customers per route. More details about these problems, in particular details relating to the characteristics of the time windows, may be found in [Solomon 1987].

The results of these experiments are summarized in Tables Ia, Ib, and Ic. In the tables, "Initial solution" refers to the solutions generated with Solomon's heuristic, using Solomon's own code. For each class of problems, we show the average number of routes, travel time, waiting time, route time and computation time (in minutes and seconds on a SPARC workstation). It is important to note that route time refers here to the sum of travel time and waiting time, and does not include the service time at each customer. In order to include the (fixed) service times, a value of 9000 must be added to the route time statistics for the problems in class C1 and C2 and a value of 1000 for the remaining classes.

In the experiments Tabu I and Tabu 2, the parameter values for the tabu search heuristic were set as follows:

Tabu 1	Tabu 2
$I_{or} = I_{2opt*} = 25$	$I_{or} = I_{2opt*} = 25$
$I_{tot} = 2500$	$I_{tot} = 5000$
$h = 6$	$h = 10$
$q = 25$	$q = 25$

We also included, for comparison purposes, the results of a classical improvement or "hill-climbing" heuristic based on the 2-opt* and Or-opt exchanges. Here, the whole 2-opt*/Or-opt neighborhood of the current solution is explored. Consequently, the exchanges do not need to satisfy "neighborhood" conditions, as in the tabu search heuristic (c.f. the *h* parameter). As soon as a feasible solution which improves the current solution is found, it is selected as the new current solution. The procedure is then repeated until no more improvement is possible.

We can see in the tables that great improvements have been achieved with the hill-climbing and tabu search heuristics on all problem classes. The hill-climbing approach was quite good at improving the initial solutions and was even better than the tabu search on problem class RC2. However, for the other classes of problems, the tabu search provided significant improvements with respect to the number of routes and route time (at the expense of additional computation time). Increasing the value of the parameters *h* and *Itot* in Tabu 2 also provided significant improvement over Tabu 1.

R1 12 problems	Number of Routes	Travel Time	Waiting Time	Route Time	Computation Time (min:sec)
Initial solution	13.5	1430.3	255.1	1685.4	
Hill-climbing	13.4	1392.9	131.5	1524.4	0:52
Tabu 1	13.2	1363.3	140.2	1503.5	6:13
Tabu 2	13.1	1330.0	118.3	1448.3	15:23

R2 11 problems	Number of Routes	Travel Time	Waiting Time	Route Time	Computation Time (min:sec)
Initial solution	3.3	1395.8	159.2	1555.0	
Hill-climbing	3.3	1328.2	83.2	1411.4	0:55
Tabu 1	3.3	1287.1	98.5	1385.6	2:37
Tabu 2	3.3	1257.0	75.8	1332.8	5:38

Table Ia Random problems

C1 9 problems	Number of Routes	Travel Time	Waiting Time	Route Time	Computation Time (min:sec)
Initial solution	10.0	952.9	146.1	1099.0	
Hill-climbing	10.0	919.2	30.6	949.8	0:20
Tabu 1	10.0	878.2	19.7	897.9	3:14
Tabu 2	10.0	869.2	15.1	884.3	11:50

C2 8 problems	Number of Routes	Travel Time	Waiting Time	Route Time	Computation Time (min:sec)
Initial solution	3.1	723.0	119.1	842.1	
Hill-climbing	3.1	687.7	3.6	691.3	0:33
Tabu 1	3.0	628.8	3.3	632.1	2:28
Tabu 2	3.0	607.5	1.1	608.6	6:01

Table Ib Clustered Problems

RC1 8 problems	Number of Routes	Travel Time	Waiting Time	Route Time	Computation Time (min:sec)
Initial solution	13.5	1596.4	175.8	1772.2	
Hill-climbing	13.4	1548.3	77.8	1626.1	1:09
Tabu 1	13.4	1521.8	81.2	1603.0	7:31
Tabu 2	13.2	1518.9	74.4	1593.3	18:23

RC2 8 problems	Number of Routes	Travel Time	Waiting Time	Route Time	Computation Time (min:sec)
Initial solution	4.0	1675.7	306.4	1982.2	
Hill-climbing	3.9	1623.7	106.4	1730.1	1:05
Tabu 1	4.0	1576.3	217.9	1794.2	2:45
Tabu 2	3.9	1575.8	156.2	1732.0	5:13

Table Ic Mixed Problems

Table II summarizes the improvement over the initial solutions with respect to the sum of travel time and waiting time. To this end, the following statistics is used:

$$\frac{\text{Initial solution - Final solution}}{\text{Initial solution}} * 100$$

	R1	R2	C1	C2	RC1	RC2
Hill climbing	9.6%	9.2%	13.6%	17.9%	8.2%	12.7%
Tabu 1	10.8%	10.9%	18.3%	24.9%	9.5%	9.5%
Tabu 2	14.1%	14.3%	19.5%	27.7%	10.1%	12.6%

Table II Improvement over Solomon's solutions

Apart from reducing route time, the tabu heuristic also saved some routes, particularly in classes R1 and RC1. It is thus clear that the tabu search is a powerful method for identifying good solutions. The exchange heuristic within the tabu search is also a very important component, if we consider that the hill-climbing heuristic in itself was quite good at improving Solomon's initial solutions.

Section 5. Conclusion

In this paper, we have introduced a tabu search heuristic for the Vehicle Routing Problem with Time windows. Important gains with respect to the total number of routes and total route time have been achieved over Solomon's initial solutions. Future developments will now be aimed at improving the method by considering the introduction of diversification techniques and, secondly, by allowing moves that generate non feasible solutions along the search path,

while keeping some control over the magnitude of violations to the time windows.

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